Don't Do Things You Can't Undo: Reversibility Models for Generating Safe Behaviours

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Abstract— We argue that an ability to determine the reversibility of actions allows a robot to identify safe behaviors autonomously. We introduce a notion of reversibility model and give a definition of model refinement. We implement this on a real robot and observe that, when a reversibility model is refined by the addition of proximity sensors, obstacle avoidance emerges as a side-effect of avoiding irreversible actions. We interpret this as evidence of a deep connection between reversibility and safe behaviour. We also observe that, on the real robot, reversibilities are learned as efficiently as a dedicated reward function. We conclude that reversibility identification may provide an abstract and yet practical method of generating a variety of safe behaviours.

I. INTRODUCTION

This paper is concerned with a robot's ability to undo its actions. We suggest that reversibility, a necessary condition of controllability, is a fundamental concept when programming robots to behave safely and reliably. We ask if this principle can be used to govern the operation of a robot, and to generate useful behaviour on a real robot and in real time.

We speculate that the most undesirable actions in the real world, those that damage the robot or the environment, for example, are characterized by irreversibility. Thus, instead of programming the robot with specific routines that prevent collisions, prevent falls, and so on, we program the robot with a more general principle of avoiding irreversible actions. In other words, instead of telling the robot *what* should not be done, we try to tell it *why* it should not be done. For example, falling down stairs is not good because the robot does not know how to climb back or pushing the door closed is not good because it does not have knowledge of how to open it.

In this paper, we state the problem of learning a reversibility model. The reversibility model represents the robot's knowledge of state-action pairs that are reversible and the ways of reversing them. We go on to demonstrate how this reversibility model can be established and used to generate new behaviours. In [1], we showed that by suppressing irreversible actions the robot will develop obstacle avoidance behaviour. In this paper, we confirm this result and go on to demonstrate that, as a developmental system, the efficiency of our abstract approach is comparable to

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An enormous amount of the robot literature is concerned with algorithms for avoiding collisions as this is considered an essential ability for mobile robots. In this literature, the goal of avoiding collisions is explicitly identified [2], while the solution may be coded for by hand or obtained indirectly using learning algorithms [3, 4]. Collision-free navigation can be learned, for example, by using genetic algorithms [5], adaptive fitness functions [6], neural networks [7] or Qlearning [8]. In [9], navigation behaviours are derived by classifying random sensor data. Our approach is different in that reliable navigation emerges from an abstract rule. The rule is not grounded in a specific sensor-motor semantics that explicitly identifies collisions, and so the resulting developmental system is insensitive to sensor permutations and inversions. Indeed, the code can be written without knowing the location or polarity of sensors and actuators, an odd sensation after years of reaching for a manual.

The idea of generating behaviours top-down from abstract principles is an emerging theme in parts of the autonomous robotics community. In developmental robotics, for example, relatively abstract emotional and motivational mechanisms are used to derive behaviours that facilitate social interaction [10, 11]. Kaplan and Odeyer show that a number of basic visual behaviours can emerge from abstract motivational principles based on prediction errors [12]. The general idea is to identify principles that can be expressed without reference to the ground meaning of sensor-motor values. Code based on such principles should function reliably in a broad range of environments and on different robots or on different parts of the same robot. Our maxim of avoiding irreversible actions is just one example of such a principle.

In the following section we present these ideas about reversibility in a more formal manner. In Section III, we describe an experimental set-up with a Khepera mini-robot that tests the reversibility principle. In Section IV, we present the results and, in the last section, we discuss these results, draw conclusions and envision possible directions for future work.

II. REVERSIBILITY MODELS

A reversibility model tells the robot which actions are reversible and how to reverse them if they are. In a fixed, known, exact, deterministic world, modelled by a graph Gof states and actions, an action from state s to state s' is reversible if there is an action back from s' to s. If we admit sequences of actions, by taking $G = \text{Path}G_0$, the graph of paths over G_0 , where G_0 is some graph of atomic actions, then finding reversibilities in G is equivalent to finding loops in G_0 , a standard problem in graph theory.

Real robots, however, face a changing, partially known, inexact and non-deterministic world. We therefore model non-determinism using labelled transition systems, we allow inexactness with a metric on the space of states, and we define a reversibility model pragmatically to be a set of expected reversibilities that may grow or shrink as the robot gains experience.

In addition, the robot may itself be changing as it learns, reconfigures or develops. In this paper we consider one form of development, the addition of sensors, and introduce a notion of refinement that captures the relationship between the robot's world before and after this development. In the learning experiments we describe, a reversibility model for an unrefined world is adapted to a refined world (with the interesting side-effect of producing obstacle avoidance behaviour).

Suppose we have a set S of states given by vectors of sensor values and a set A of actions given by vectors of motor commands. If we view the states as the nodes in a graph and the actions as labels, the robot's body and environment determine a labelled transition system which we refer to as the robot's *world*. A labelled transition system is a standard structure for modelling non-deterministic systems and consists of a directed graph with edges, called transitions, labelled by actions. When the result of an action a in state s is not wholly determined by the robot, multiple transitions from s are labelled with the same action a and it is the world that determines which transition actually happens.

A reversibility for a world W is a state-action pair (s, a), together with a state-action pair (s', \bar{a}) . A reversibility may or may not hold, in a mathematical sense or in a physical sense. Generally speaking, \bar{a} is expected to produce a transition from s' to s, assuming a produces a transition from s to s' in W. Because of the non-determinism, even given a perfectly known world W, there are different ways to define 'holding'. A reversibility $((s, a), (s', \bar{a}))$ may hold weakly if there exists in W a transition from s to s' labelled a and a transition from s' to s labelled a. Or, it may hold strongly if there exists a transition from s to s' labelled a and every transition from s' labelled \bar{a} , and at least one, leads to s. In our implementation, we use the strong definition. In addition, the action \bar{a} is expected to work for any state x with $d(x, s') < \epsilon'$ and is only expected to produce a transition back to a state ywhen $d(y,s) < \epsilon$, where d is a metric on states.

A reversibility model for a world W is a set of reversibilities for W that are expected to hold. In practice, a

reversibility model could be given in advance, communicated to the robot, learned empirically, deduced from knowledge about the world, or obtained in some other way. In the experiments described here, the robot is given a model for one world and uses this to learn a model for a refined world.

A refinement (of states) from a world W to a world W' is a pair of functions from the states and transitions of W' back to those of W, that respects the graph structure and labelling and is surjective on states. In other words, every state in W is the image of one or more states in W', which 'refine' the state in W, and the action on an edge in W' is given by the action on the edge it is sent to in W.

For any reversibility model R for a world W and for any refinement from W to W', with state function p, there is a refined set of reversibilites R' on W' defined by

$$R' = \{((s,a), (s',\bar{a})) | ((p(s),a), (p(s'),\bar{a}) \in R\}.$$

To obtain a reversibility model for the new world W' we may form R' and then remove any pairs that fail in the refined world. An important aspect of this procedure is that 'it gives the robot something to do': the original model R provides a specific list of actions together with the circumstances in which they should be tried.

The kind of refinement we have in mind is produced by extending a robot's sensor vector. Suppose we have a world with states given by pairs of wheel counter values (w_1, w_2) and actions given by pairs of wheel displacement commands (m_1, m_2) . Assuming the robot is able to control its own wheels, this world is fairly deterministic, all actions are reversible and a good reversibility model R is given by taking $\bar{a} = (-m_1, -m_2)$ when $a = (m_1, m_2)$ (for any s and s').

Now suppose we include one proximity value (say, the front sensor) in the state vector (w_1, w_2, d_1) . Assuming the new sensor does not effect the robot's environment, we obtain a refinement of the original world. The state function p is the projection

$$p(w_1, w_2, d_1) = (w_1, w_2).$$

When the simple model R described above is refined according to this new world some of the refined reversibilities hold and some do not. In our experiments, the robot tests these refined reversibilities to discover which hold.

The interesting point here is that the ones that fail generally correspond to collisions of some sort. Consider the following four cases (in which wheel counts and proximities are given, without loss of generality, in comparable units).

1) The robot does not touch anything: we obtain, say, the successful reversibility

(((0, 0, 15), (10, 10)), ((10, 10, 5), (-10, -10))),

where the robot approaches and retreats from an object without touching it.

- 2) The robot touches an object and the object slides: we obtain a failed reversibility, say
 - (((0, 0, 8), (10, 10)), ((10, 10, 0), (-10, -10))),

where the robot runs into an object, pushing it 2 units forward, and then retreats, only to find that, while its wheel encoders are back to 0 as expected, its proximity sensor now reads 10 instead of the original 8.

- 3) The robot runs into an object and its wheels slide: from the robots point of view, this is identical to Case 2.
- 4) The robot runs into an object and its motors stall: if motor commands time-out and report success, adjusting the wheel encoder counts as necessary, then this case is again identical to Case 2 (and may be thought of as a kind of internal sliding).

Not only does the robot discover that it is 'bad' to push things-without ever knowing what pushing is!-but the refined state allows the robot to distinguish those cases in which 'bad things happen' from those in which they do not. Once the robot learns a reversibility model, it may use the model to censor its actions. Because of the nondeterminism, we have a growing choice of definitions. A state-action pair (a, s) is strongly reversible in world W, if there is a reversibility $((s, a), (s', \bar{a}))$ that holds in W for every s' that can be reached from s by a transition labelled a. Alternatively, we could ask for just one such s'to get a definition of *weakly reversible*. We must also say if $((s, a), (s', \bar{a}))$ holds strongly or weakly in W, for a total of four definitions. In our experiments, we use, in effect, the strong-strong definition, but because we pretend the world is deterministic by ignoring s (by taking $\epsilon' = \infty$), there is no real difference.

Note that it is our method of creating a reversibility model out of R' by pruning that creates a pushing-is-bad model. Alternatively, when a reversibility $((s, a), (s', \bar{a}))$ in R' fails, we could try replacing the action \bar{a} instead of throwing out the reversibility. For example, we could construct the world $W'^* = \text{Path}W'$. The transitions in PathW' are paths of transitions in W' labelled by sequences of actions from W'. The world W' embeds in W'^* , along with R', but now we have sequences of actions to play with. In the object pushing example, a sequence b of actions might cause the robot to go behind an object, push it back 2 units, and then return to its original place in front of the object, so that

$$(((0, 0, 8), (10, 10)), ((10, 10, 0), b)),$$

holds in W'^* . Or we could form W'^* by adding a gripping action and simply drag the object back 2 units.

III. EXPERIMENTS

This section describes experiments with two learning algorithms. In all the experiments, both algorithms learn from the same sequence of actions and sensor data. One learns which reversibilities hold or fail. The other one is a standard reinforcement algorithm that punishes collisions. We compare the performance of the two algorithms over four sequences of actions. These were produced by running the same action generation routine in two test environments, an easy one and a harder one, and over two sets of actions, 1D and 2D. The experiments were conducted on a Khepera II minirobot, which is a cylindrical robot about 7 cm in diameter (see Fig. 1) with differential drive and a ring of eight proximity sensors. In these experiments, the motor control parameters were set so that, when the robot runs into a wall, the motors stall before the wheels slip. This allows us to detect collisions by watching for stalled motors. When a collision does happen, the wheel command routine times out and reports success, up-dating the wheel counters as if the command had completed. This is equivalent to more a forceful wheel command that would cause the wheels to slip, but makes it easier to identify collisions, which is required for the reinforcement algorithm and used for evaluating both algorithms.

A. Implementation Details

An action $a = (m_1, m_2)$ consists of a pair of motor displacement commands, for left and right wheels, expressed in native wheel decoder units. A discrete set of actions is used in the experiments:

a_1	=	(100, 100) short step forward,
a_2	=	(300, 300) long step forward,
a_3	=	(-100, -100) short step backward,
a_4	=	(-300, -300) long step backward,
a_5	=	(100, -100) rotate clockwise,
a_6	=	(-100, 100) rotate conterclockwise.

In the 1D experiments, we take

$$A = \{a_1, a_2, a_3, a_4\}.$$

These actions cause the robot to move back and forth in a straight line. In the 2D experiments, we include the turning actions,

$$A = \{a_1, a_2, a_3, a_4, a_5, a_6\}.$$

We provide the robot with the initial reversibility model

$$\{((x, a_1), (x + (100, 100), a_3)), \\ ((y, a_2), (y + (300, 300), a_4)), \\ ((z, a_5), (z + (100, -100), a_6))\},$$

where x, y and z are any states (w_1, w_2) , consisting of a pair of wheel counter values. Because we have fixed things so that wheel commands always succeed, the reversibilities in this model always hold. We then use (in effect) a refinement function p, the projection from the set of states $(w_1, w_2, d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8)$, which include eight proximity values, to the original set of states (w_1, w_2) without the proximity values, to induce a new set of refined reversibilities from the original set. The new set contains, for example,

$$\begin{aligned} &((s,a_1),(s',a_3)) = \\ &(((w_1,w_2,d_1,d_2,d_3,d_4,d_5,d_6,d_7,d_8),a_1), \\ &((w_1+100,w_2+100,d_1',d_2',d_3',d_4',d_5',d_6',d_7',d_8'),a_3)) \end{aligned}$$

for any w_i , d_i and d'_i . The learning algorithm then tests these to see which hold and which fail.

For our definition of 'near', we use the Manhattan metric defined by

$$d(s,s') = \sum_{i=1}^{2} |w_i - w'_i| + \sum_{i=1}^{b} |d_i - d'_i|$$

(but because our wheel commands always succeed, and the original model is correct, the wheel value part of this is always 0).

a) Robot motion: The Khepera runs in a real, physical environment with motions that test the pairs of the refined reversibility model. The robot moves according to the following algorithm:

- 1) Record current state $s_i = (w_1, w_2, d_1, ..., d_8)$.
- 2) Choose an arbitrary reversibility from R' and execute the forward action as a_i .
- 3) Record the state $s_{i+1} = (w'_1, w'_2, d'_1, \dots, d'_8)$.
- 4) Execute the reverse action as a_{i+1} .
- 5) Record the resulting state as s_{i+2} .
- 6) Execute a random action as a_{i+2} .
- 7) Add 3 to *i* and repeat.

So the robot performs a random forward action, then the supposed reverse action, then a random action that goes unreversed, and then another forward action, and so on.

b) Learning the reversibility model: As the robot moves about, it notes how well the reversibilities hold using the Manhattan metric.

For each forward action a_i , calculate and store $d(s_i, s_{i+2})$.

For the purposes of comparison with the reinforcement algorithm, the model is also used to predict which actions will be successfully reversed. When a failure is predicted, we note whether there is a collision during the action. So we are judging the reversibility model not by what it is meant to be learning, but by how well this happens to predict collisions.

- 1) Get the current state s_i and the intended action a_i
- 2) From memory, choose a state-action pair (s_k, a_i) that minimizes $d(s_k, s_i)$.
- If we have d(s_k, s_i) > δ, predict randomly. Otherwise, predict a collision unless d(s_k, s_{k+2}) < ε.
- 4) While executing the command a_i check if there is a collision. Store the predicted and the actual outcome.

c) Reinforcement learning: Reinforcement learning algorithms [13] are commonly used in mobile robotics. The aim here is to implement a simple version for collision avoidance so that we may compare the ungrounded reversibility method to a standard, grounded method. We have therefore implemented the reinforcement learning algorithm so that the robot is operating under similar conditions. First, the algorithm does not have a terminal state, so collision avoidance is considered to be a continuous task of reward maximization. Second, the current version of the reversibility policy is concerned only with immediate actions and reverse actions and does not work along the history of action sequences.



Fig. 1. The robot in Environment I and in Environment II.

Therefore we have also implemented the reinforcement algorithm to be concerned only with immediate rewards, thus with discount rate $\gamma = 0$. The initial value of the action value function is $Q(s_i, a_i) = 0$. The reward signal is defined by checking for collisions.

$$r = \frac{(|w_1| + |w_2|)/100}{-5}$$
, if there is a collision

Thus a successful action is rewarded more if it moves the robot a greater distance and an unsuccessful action is strongly penalised. Note that the reinforcement learning algorithm directly checks for collisions (by watching for stalled motors) to calculate the reward, while the algorithm learning the reversibility model only aims at predicting if the robot can return to the initial state (by watching the proximity sensors). The reinforcement learning algorithm is the following.

- 1) Get the current state s_i and the intended action a_i .
- 2) If the current value of the action value function $Q(s_i, a_i) < 0$, predict a collision. If $Q(s_i, a_i) = 0$ make a random prediction. Otherwise, predict no collision.
- 3) After executing a_i get the reward signal r.
- 4) Update the action value function

predicted and the real outcome.

 $Q(s_i, a_i) \leftarrow \alpha r + Q(s_i, a_i)$, with learning rate $\alpha = 0.1$. 5) While executing a_i , check for collisions. Store the

B. Test Environments

In the experiments we compared the learning of reversibility models to the learning of a reward function that discourages collisions. To find out how sensitive the learning algorithms are to environmental conditions, the tests are conducted in two environments. These are shown in Fig. 1. Environment I is a rectangular space, whereas Environment II is a smaller, triangular space, only slightly larger than the robot, in which collisions are more probable. In both environments the algorithms are run over sequences of 1D movements and sequences of 2D movements. With 1D actions, the robot only moves forewards and backwards. With 2D actions, the robot moves in all directions. This was done



Fig. 2. Experimental results in Environment I, with 1D actions.



Fig. 3. Experimental results in Environment I, with 2D actions.

to get an idea of how the algorithms scale from smaller to larger, more complex action sets.

IV. RESULTS

As described in the previous section, the robot operates by executing supposed reversals and random actions. The reversed actions are determined according to the initial reversibility model. The goal of the learning algorithms is to observe and learn to predict the outcomes of actions. These predictions are then compared to the real outcome of the action (determined by detecting collisions) and the success rate of each method is recorded.

Note that although the performance graphs for the two methods are expressed in the same terms, we are *not* comparing two techniques for solving one learning problem, but rather two learning problems whose solutions happen to result in the same behaviour. The reversibility problem is at a big disadvantage here, because we are evaluating it as if it was intended to predict collisions, which is in fact just a fortuitous emergent property.

Moreover, the motion routine, which performs reversals interleaved with random actions, allows a reversibility to be tested every third step, while the reinforcement algorithm gets a feedback signal at every step. Thus the reinforcement learning algorithm has more experiences to learn from, and



Fig. 4. Experimental results in Environment II, with 1D actions.



Fig. 5. Experimental results in Environment II, with 2D actions.

yet the performance of the two algorithms is seen to be comparable.

The figures show the performance of the two algorithms over four sequences of actions and sensor values. All four graphs show the average correctness of predictions for each successive 100 actions for both prediction methods. From Fig. 2 and Fig. 4, we see that with 1D actions the robot rapidly learns to avoid collisions in both environments. The rate of successful predictions reaches 80during the first 200–300 steps, and the learning problem is equally trivial for both learning algorithms. From Fig. 3 and Fig. 5, we see that with 2D actions the learning problems are more complicated, with both algorithms converging around 1900–2100 steps.

During the runs in Environment II, the wheels occasionally got stuck on the uneven surface. These incidents can be seen on the graphs around 1400–1700 steps in 1D (Fig. 4) and 3700–4100 in 2D (Fig. 5), where there are sharp downward peaks in the prediction rates. It appears that reinforcement learning recovers better. However, this is caused more by the method we use to determine the prediction rate than a failure to relearn the reversibility model. For the robot with the blocked wheels, the reversibility of actions is perfect, since the robot certainly ends up in the same state it starts from. However, when this is used to predict no collision, which is to say no motor stall, the prediction is wrong.

In these runs, which consist of thousands of actions, it is

clear that both learning problems are solved with comparable speed. The reversibility model is learned with roughly the same speed as the reward function in Environment I, whereas in Environment II reinforcement learning happens slightly faster. Likewise, both approaches scale equally well from a 1D to a 2D environment.

V. CONCLUSIONS

This paper introduces the concept of reversibility for learning and developing robots. We show that reversibility models can be used to learn a useful new behaviour. The experiments verify the performance of the reversibility method against a well-established learning method commonly used in robotics. The results show that both of the methods converge to obstacle avoidance behaviour.

The most general conclusion drawn from the experimental results is that the efficiency of the policy of reversibility is comparable to reinforcement learning. Both methods learn more or less equally, converging to satisfactory performance. The basic difference of these methods is that the reinforcement learning algorithm uses a reward signal explicitly designed to make the robot avoid obstacles. The policy we introduce, uses a reversibility measure to learn a reversibility model, and yet the robot learns the useful behaviour of collision avoidance.

Based on these experimental results we speculate that the concept of reversibility could generate a variety of useful behaviours depending on the properties of the environment. We surmise, for example, that a robot placed initially close to an object or wall might, using reversibility models, discover behaviours like 'do not leave the territory' or 'stay in the vicinity of guidelines'. Our future experiments are planned to check this hypothesis and find more evidence concerning the robustness of this principle.

Another hypothesis we are planning to test is whether learning algorithms can be accelerated by using reversibility models. Generally, learning algorithms converge to a stable behaviour by repeating actions that lead from one state to another. The problem of how the robot gets back to the state it wants to repeat, however, is not addressed. Knowing the reversibility model, it may be easier to guide the learning algorithm to faster convergence. We also suggest that reversibility models could be used in combination with formal reasoning methods, such as task or path planning, where the plans can be checked for reversibility. For mobile robots such a reversibility check could, for example, guarantee safe homing or safe exploration. We suggest that the concepts introduced in this paper may provide handy and simple guidelines for building safe and reliable robots.

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